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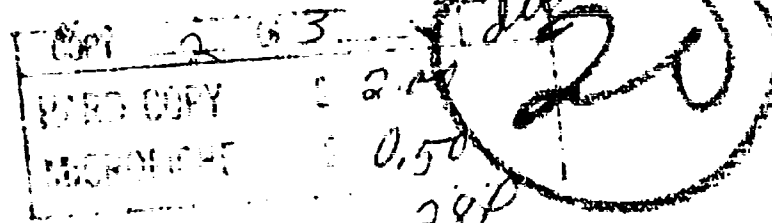
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Linear Van Atta reflector consisting
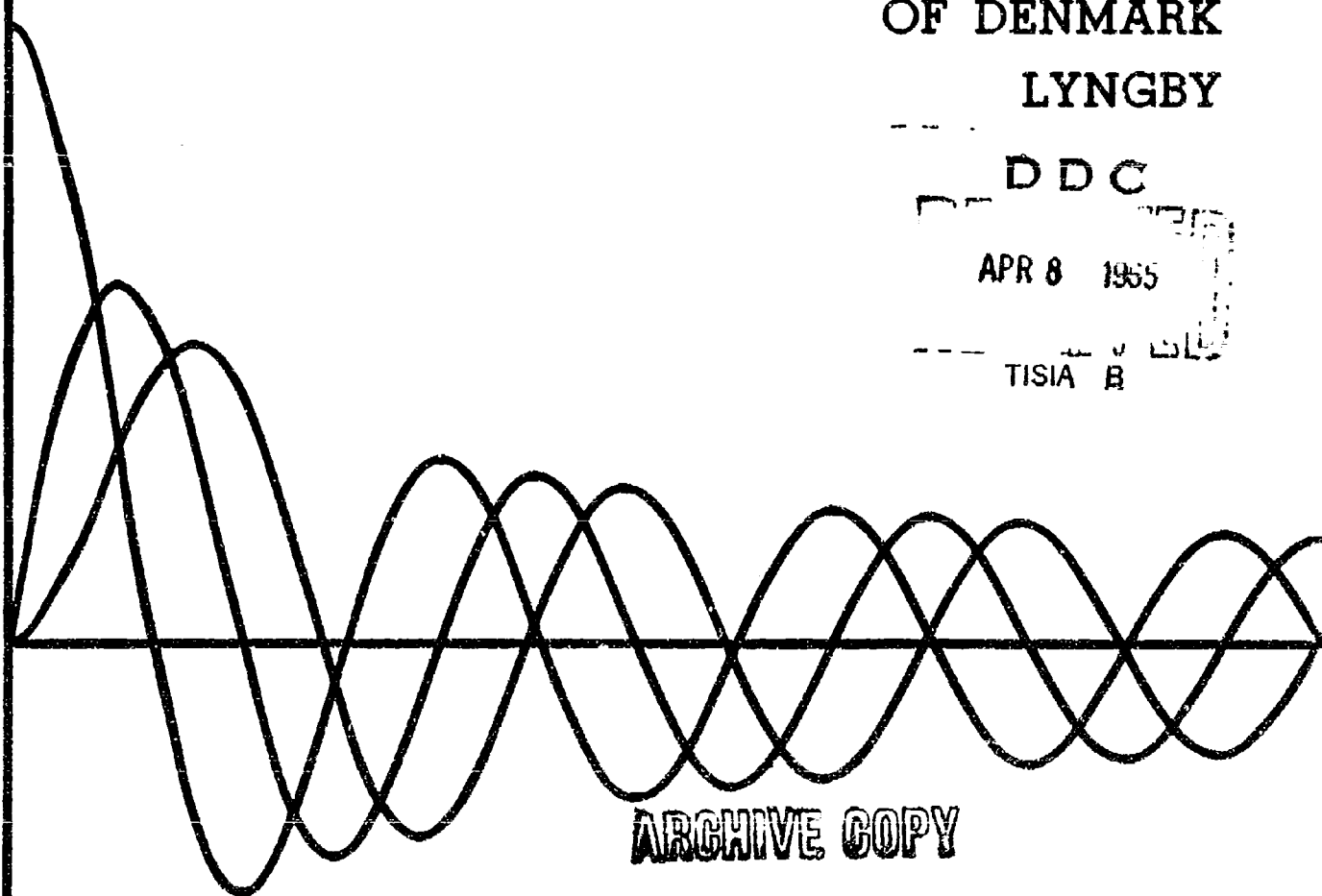of four half-wave dipoles

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1. INTRODUCTION

Since Van Atta(1) proposed his reflector in 1956 many papers have been written on reflectors of this type. A survey of these papers was given in Scientific Report No. 1, in which a scheme was proposed for a theoretical investigation of an arbitrary Van Atta reflector. It was shown how the differential scattering cross section may be calculated. An equivalent diagram for each pair of antennas was set up using the general X-circuit to represent the transmission lines connecting the dipoles. The equivalent diagrams take into account the scattering effect of the antennas as well as the coupling between the antennas.

In this report the transmission lines are considered as phase shifters. It turns out that an analytic investigation is more easy to perform when the transmission lines are considered as phase shifters than when an equivalent X-circuit is used.

The reflected field will be found by superposition of three fields: 1. the field reflected due to the interconnection between the antennas, 2. the field due to the scattering effect of the antennas, and 3. the field due to the coupling between the antennas.

In the former Air Force report a few numerical calculations were made for a linear Van Atta reflector consisting of four half-wave dipoles. It is the purpose of this report to make an analytic and numerical investigation of this simple reflector in order to obtain results which indicate the behaviour of an arbitrary Van Atta reflector.

2. GENERAL CONSIDERATIONS

Before formulating an expression for the reflected field some general considerations will be made in this section. A linear Van Atta reflector consisting of four antennas is shown in fig. 1. Each pair of antennas is connected by transmission lines of equal length. When a plane wave is incident on the reflector a current distribution will be generated in the antennas. If we do not take into account the coupling between the antennas this current distribution may be divided into two parts. The first part is the current generated in each antenna by the incident wave. The second part is due to the interconnections between the antennas, since each antenna receives energy from the incident wave and this energy is transmitted through the transmission line to its mate. Let the direction of propagation of the incident plane wave make an angle ϕ_i with respect to the plane of the reflector. In fig. 2 it appears that the fields from the first part of the above-mentioned currents are in phase in a direction making an angle $\pi - \phi_i$ with respect to the plane of the reflector and in fig. 1 it appears that the second part of the currents set up field which are in phase in the direction back in the direction of arrival of the incident wave because of the equal length of the transmission lines.

The field set up in fig. 2 is due to the scattering effect of a receiving antenna and the field set up in fig. 1 is the field considered in Van Atta's patent description (1).

In the patent description it is claimed that the reflector reradiates maximum energy back in the direction of arrival. Since the field reradiated due to the scattering effect is of the same order of magnitude (see Appendix) as the field considered in the patent-description it turns out that if the reflector works as stated in the patent description it also operates as a mirror. However, on account of the interference between the two fields it appears that we cannot always expect the effect mentioned in the patent description. Furthermore, if the angle of incidence and the length of the transmission lines is such that the two parts of the current on each element are in opposite phase we will get no reflection at all.

As could be expected it appears from the numerical investigation of the special reflector that the mutual impedances may increase or decrease the reflection back in the direction of arrival or the mirror effect depending on the distance between the antennas, the length of the transmission lines, and the angle of incidence.

The above considerations are valid in general, since the four anten-

nas in fig. 1 may be considered as two pairs of antennas in an arbitrary Van Atta reflector.

3. THE REFLECTED FIELD

The reflected field will now be determined when the antennas in fig.1 are half-wave dipoles. The distance between the dipoles is $p \cdot \lambda$, where λ is the wavelength and p is a real number. The length of the transmission lines are ℓ. In the general considerations given above the direction of incidence was arbitrary. Here the reflected field will be determined when the direction of propagation of the incident wave is in the plane normal to the axis of the dipoles making an angle ϕ_i with respect to the plane of the reflector. The plane wave will generate the total currents I_1, I_2, I_3 and I_4 in the four dipoles named 1, 2, 3 and 4, respectively.

Each of these currents may be divided into four currents. Consider the equivalent diagram in fig. 3 for dipole 1 and 4. The propagation constant in the transmission lines is β. By means of transforming network the antenna impedances Z_{An} are matched to the impedance of the transmission lines. V_1 and V_4 are the free space open-circuit voltages induced by the plane wave in dipole 1 and 4, respectively. The other voltages shown in fig. 3 are due to the coupling between the antennas. Z_{12}, Z_{13}, and Z_{14} are the mutual impedances between half-wave dipoles with a distance of $p\lambda$, $2p\lambda$ and $3p\lambda$, respectively.

The four currents I_{11}, I_{12}, I_{13}, and I_{14}, which compose I_1 will be given by,

1. $\quad I_{11} = \dfrac{V_1}{2R_{An}}$, due to V_1 on dipole 1.

R_{An} is equal to the real part of Z_{An}.

2. $\quad I_{12} = - \dfrac{V_4}{2R_{An}} e^{i\beta\ell}$, due to V_4 on dipole 4,

which, through the transmission line, generates a current in dipole 1.

3. $\quad I_{13} = \dfrac{-Z_{12}I_2 - Z_{13} I_3 - Z_{14}I_4}{2R_{An}}$, due to the coupling between dipole 1 and

dipoles 2, 3, and 4.

4. $\quad I_{14} = - \dfrac{-Z_{14}I_1 - Z_{13}I_2 - Z_{12}I_3}{2R_{An}} e^{i\beta\ell}$, due to the coupling between dipole 4

and dipoles 1, 2, and 3.

The time factor is $e^{-i\omega t}$, and the current distribution is assumed to be sinusoidal. I_{11} represents the scattering effect of dipole 1. I_{12} represents the current dealt with in Van Atta's patent description. It appears that I_{11} has the same magnitude as I_{12}, and that I_{13} and I_{14} are not negligible to I_{11} and

I_{12}., when the mutual impedances are comparable to R_{An} as here, where the distances between the antennas do not exceed a few wavelengths.

Corresponding consideration for the other dipoles give

$$(2R_{An}-Z_{14}e^{i\beta\ell})I_1 + (Z_{12}-Z_{13}e^{i\beta\ell})I_2 + (Z_{13}-Z_{12}e^{i\beta\ell})I_3 + Z_{14}I_4 = V_1-V_4 e^{i\beta\ell} \quad , \quad 1.$$

$$(Z_{12}-Z_{13}e^{i\beta\ell})I_1 + (2R_{An}-Z_{12}e^{i\beta\ell})I_2 + Z_{12}I_3 + (Z_{13}-Z_{12}e^{i\beta\ell})I_4 = V_2-V_3 e^{i\beta\ell} \quad , \quad 2.$$

$$(Z_{13}-Z_{12}e^{i\beta\ell})I_1 + Z_{12}I_2 + (2R_{An}-Z_{12}e^{i\beta\ell})I_3 + (Z_{12}-Z_{13}e^{i\beta\ell})I_4 = V_3-V_2 e^{i\beta\ell} \quad , \quad 3.$$

$$Z_{14}I_1 + (Z_{13}-Z_{12}e^{i\beta\ell})I_2 + (Z_{12}-Z_{13}e^{i\beta\ell})I_3 + (2R_{An}-Z_{14}e^{i\beta\ell})I_4 = V_4-V_1 e^{i\beta\ell} \quad , \quad 4.$$

These equations may be transformed to

$$R_{An}(1+i\cot\beta\ell)I_1 + Z_{12}I_2 + Z_{13}I_3 + (i\frac{R_{An}}{\sin\beta\ell} + Z_{14})I_4 = V_1 \quad , \qquad 5.$$

$$Z_{12}I_1 + R_{An}(1+i\cot\beta\ell)I_2 + (i\frac{R_{An}}{\sin\beta\ell} + Z_{12})I_3 + Z_{13}I_4 = V_2 \quad , \qquad 6.$$

$$Z_{13}I_1 + (i\frac{R_{An}}{\sin\beta\ell} + Z_{12})I_2 + R_{An}(1+i\cot\beta\ell)I_3 + Z_{12}I_4 = V_3 \quad , \qquad 7.$$

$$(i\frac{R_{An}}{\sin\beta\ell} + Z_{14})I_1 + Z_{13}I_2 + Z_{12}I_3 + R_{An}(1+i\cot\beta\ell)I_4 = V_4 \quad , \qquad 8.$$

when $\sin\beta\ell \neq 0$.

If V_t is the open-circuit voltage induced in a reference antenna placed at the center of the reflector, we have

$$V_1 = V_t e^{i3x_i} \qquad , \qquad\qquad 9.$$

$$V_2 = V_t e^{ix_i} \qquad , \qquad\qquad 10.$$

$$V_3 = V_t e^{-ix_i} \qquad , \qquad\qquad 11.$$

$$V_4 = V_t e^{-i3x_i} \qquad , \qquad\qquad 12.$$

where

$$x_i = \frac{1}{2} kp\, \lambda\cos\phi_i = \pi \cdot p \cdot \cos\phi_j. \qquad\qquad 13.$$

k is the free-space propagation constant. The electric fieldstrength at a point in the plane normal to the axes of the dipoles, at a distance r from the center of the array in a direction making an angle ϕ_n with respect to the plane of the reflector is (2)

$$E = \frac{-i\xi}{2\pi} \frac{e^{ikr}}{r} (I_1 e^{i3x_u} + I_2 e^{ix_u} + I_3 e^{-ix_u} + I_4 e^{-i3x_u}) , \qquad\qquad 14.$$

where ξ is the characteristic impedance of free space and

$$x_u = \frac{1}{2} kp \cdot \lambda \cdot \cos\phi_u = \pi \cdot p\cos\phi_u . \qquad\qquad 15.$$

Solving 5., 6., and 8. and using 9., 10., 11., and 12. we find, for the reflected field

$$E = \frac{-iV_t\xi}{\pi} \frac{e^{ikr}}{r} \frac{1}{2D}$$

$$\{(D_1 e^{i3x_i} + D_2 e^{ix_i} + D_3 e^{-ix_i} + D_4 e^{-i3x_i})\, e^{i3x_u}$$

$$+ (D_2 e^{i3x_i} + D_5 e^{ix_i} + D_6 e^{-ix_i} + D_3 e^{-i3x_i})\, e^{ix_u}$$

$$+ (D_3 e^{i3x_i} + D_6 e^{ix_i} + D_5 e^{-ix_i} + D_2 e^{-i3x_i})\, e^{-ix_u}$$

$$+ (D_4 e^{i3x_i} + D_3 e^{ix_i} + D_2 e^{-ix_i} + D_1 e^{-i3x_i})\, e^{-i3x_u} \}, \qquad\qquad 16.$$

where

$$D = Z_1 D_1 + Z_{12} D_2 + Z_{13} D_3 + Z_4 D_4 . \qquad\qquad 17.$$

$$D_1 = \begin{vmatrix} Z_1 & Z_2 & Z_{13} \\ Z_2 & Z_1 & Z_{12} \\ Z_{13} & Z_{12} & Z_1 \end{vmatrix} , \qquad D_5 = \begin{vmatrix} Z_1 & Z_4 & Z_{13} \\ Z_4 & Z_1 & Z_{12} \\ Z_{13} & Z_{12} & Z_1 \end{vmatrix} \qquad 18.\text{ and } 19.$$

$$D_2 = \begin{vmatrix} Z_{12} & Z_4 & Z_{13} \\ Z_2 & Z_{12} & Z_1 \\ Z_{13} & Z_1 & Z_{12} \end{vmatrix} , \qquad D_3 = \begin{vmatrix} Z_{13} & Z_4 & Z_{12} \\ Z_2 & Z_{13} & Z_1 \\ Z_{12} & Z_1 & Z_{13} \end{vmatrix} \qquad 20.\text{ and } 21.$$

$$D_4 = \begin{vmatrix} z_4 & z_{13} & z_{12} \\ z_{13} & z_2 & z_1 \\ z_{12} & z_1 & z_2 \end{vmatrix} \qquad D_6 = \begin{vmatrix} z_2 & z_{13} & z_{12} \\ z_{13} & z_4 & z_1 \\ z_{12} & z_1 & z_4 \end{vmatrix} \qquad \text{22 and 23.}$$

$$z_1 = z_{An}(1 + i \cot\beta\ell) , \qquad\qquad\qquad 24.$$

$$z_2 = i \frac{z_{An}}{\sin\beta\ell} + z_{12} , \qquad\qquad\qquad 25.$$

$$z_4 = i \frac{z_{An}}{\sin\beta\ell} + z_{14} . \qquad\qquad\qquad 26.$$

In particular in the direction back in the direction of arrival, $x_u = x_i$,

$$E = - \frac{iV_t \xi}{\pi} \frac{e^{ikr}}{r} \frac{1}{D}$$

$$\cdot (D_4 + D_6 + (2D_3 + D_5)\cos 2x_i + 2D_2 \cos 4x_i + D_1 \cos 6x_i) . \qquad 27.$$

4. THE GENERAL FORM OF THE RADIATION PATTERN.

In order to study the general form of the radiation pattern we neglect the mutual impedances in this section. The currents in the dipoles are then

$$I_1 = \frac{V_1}{2R_{An}} - \frac{V_4}{2R_{An}} e^{i\beta\ell} , \qquad\qquad 28.$$

$$I_2 = \frac{V_2}{2R_{An}} - \frac{V_3}{2R_{An}} e^{i\beta\ell} , \qquad\qquad 29.$$

$$I_3 = \frac{V_3}{2R_{An}} - \frac{V_2}{2R_{An}} e^{i\beta\ell} , \qquad\qquad 30.$$

$$I_4 = \frac{V_4}{2R_{An}} - \frac{V_1}{2R_{An}} e^{i\beta\ell} , \qquad\qquad 31.$$

In this case the electric field is

$$E = -\frac{i\xi}{2\pi} \frac{e^{ikr}}{r} \frac{V_t}{R_{An}} \cdot g , \qquad\qquad 32.$$

where

$$g = \cos 2(x_i + x_u) \cos (x_i + x_u) - \cos 2(x_i - x_u) \cos (x_i - x_u)e^{i\beta\ell} \qquad 33.$$

In the patent description it is claimed that the refelctor reradiates maximum energy back in the direction of arrival, when the transmission lines are of equal length.

From the absolute value of g we deduce below some results from which it appears that the reflector has not the effect stated in the patent description.

1. The absolute value of g is the same for $\phi_u = v$ and $\phi_u = \pi - v$. This means that the radiation pattern is symmetrical about the normal to the reflector. This indicates that if the reflector has a maximum of reradiation back in the direction of arrival, it also works as a mirror as mentioned in section 2.

2. However, the maximum reflection often is not back in the direction of incidence.

3. The absolute value of g depends on the length of the transmission lines.

4. At some lengths of the transmission lines there is no reflection at all.

As an illustration of these results we will consider the radiation pattern in the following special cases when the distance between the antennas is a half wavelength.

On account of the symmetry we will only consider ϕ_i and ϕ_u in the interval 0 to $\pi/2$.

Since we do not take into account the disturbance from the transmission

lines the radiation patterns are symmetrical about the plane of the reflector.

a. Grazing and normal incidence.

Illustrations may be found in figs. 4 and 5.

At grazing incidence we have ($\phi_i = 0$)

$$g = (\sin 3x_u - \sin x_u)(1 + e^{i\beta\ell}) , \qquad 34.$$

and at normal incidence ($\phi_i = \pi/2$)

$$g = (\cos 3x_u + \cos x_u)(1 - e^{i\beta\ell}) . \qquad 35.$$

It is seen that the form of the radiation pattern is independent of the length of the transmission lines.

At grazing incidence maximum reflection is obtained when ℓ is a whole number of wavelengths and at normal incidence when ℓ is an odd number of half wavelengths. In these two cases maximum reflection is in the direction opposite the direction of arrival when we do not take into account the mirror effect.

There are two smaller maxima at $\phi_u = 74^\circ.5$ and at $\phi_u = 42^\circ.9$, respectively. From 34 and 35 it appears that there is no reflection at all when ℓ is an odd number of half wavelengths at grazing incidence and when ℓ is a whole number of wavelengths at normal incidence. The result that there is no reflection at normal incidence when ℓ is a whole number of wavelengths is valid for an arbitrary reflector, since the right hand sides of the equations 1., 2., 3., and 4. are equal to zero.

b. Cases where only the phase of the reflected field depends on the length of the transmission lines.

When $\phi_u = \arccos(\cos\phi_i \pm 1)$,

we have

$$g = \cos 2(x_i + x_u) \cos (x_i + x_u),$$

and when $\phi_u = \arccos(\cos\phi_i + \frac{1}{2} m)$, where $m = \pm 1, \pm 3,$.

we have

$$g = - \cos 2(x_i + x_u) \cos(x_i + x_u) e^{i\beta\ell} .$$

In the first case ℓ has no influence at all, but in the last case the phase depends on ℓ.

For example when $\phi_i = 30^\circ$ there is no change in the magnitude of the reflected field in the directions $\phi_u = 50^\circ.7$, $68^\circ.5$ and $82^\circ.3$, when ℓ is changed. In fig. 6 the radiation patterns are shown when $\phi_i = 30^\circ$ and ℓ is varied.

c. **The direction of maximum reflection when ℓ is a half wavelength and when ℓ is a whole wavelength.**

When the length of the transmission lines is equal to a half wavelength we always have a maximum at $\phi_u = 90^0$ and in another direction determined by

$$\sin x_u = \sqrt{\frac{9\cos 3x_i + \cos x_i}{12 \cos 3x_i}} \, ,$$

if ϕ_u exists in the above equation.

Which one of the maxima is the greatest depends on the direction of incidence (see fig. 4). In fig. 7 ϕ_{umax}, the direction of the greatest maximum, is shown as a function of ϕ_i.

When ℓ is equal to one wavelength the two maxima are at $\phi_u = 0$ and in the direction determined by

$$\cos x_u = \sqrt{\frac{9\sin 3x_i - \sin x_i}{12 \cos 3x_i}} \, ,$$

if ϕ_u exists in the above equation.

The figures illustrating this case are fig. 5 and fig. 8.

From this it is seen that the reflector does not act at all as stated in the patent.

It should be mentioned that at 0.25λ (or 0.75λ) the deviation of ϕ_{umax} from ϕ_i is not as great as in the cases mentioned above (see figs. 9 and 10). In fact, the numerical calculations show that at these lengths we have the smallest deviation.

It is shown later on that the coupling between the antennas makes a small change in the optimum transmission line length (see section 6).

d. **The reradiation patterns are the same for $\ell = \lambda/2 + \alpha\lambda$ and $\ell = \lambda/2 - \alpha\lambda$.**

It can be shown from the absolute value of g that the radiation pattern is the same for $\ell = \lambda/2 + \alpha\lambda$ and $\ell = \lambda/2 - \alpha\lambda$, where α is a real number.

5. THE INFLUENCE OF COUPLING BETWEEN THE DIPOLES.

In order to illustrate the influence of coupling between the dipoles reradiation patterns are calculated when the mutual impedances are taken into account. In figs. 4, 5, 9, and 10, it is possible to compare reradiation patterns where the mutual impedances are neglected with reradiation patterns where the impedances are taken into account.

It was mentioned above that the reradiation pattern are the same for $\ell = \lambda/2 + a\lambda$ and $\ell = \lambda/2 - a\lambda$ when coupling is neglected. In figs. 9 and 10 the reradiation patterns are shown for two such line lengths, namely $\ell = 0.25\lambda$ and $\ell = 0.75\lambda$. It is to be noted that the radiation pattern is not symmetrical about the normal to the reflector.

However in the cases where ℓ is a multiple of half wavelengths, there is symmetry. This may be seen from equations 1. to 4., and is illustrated in figs. 4 and 5.

The diagrams show that the coupling does not effect the principal form of the patterns but it appears that the induction may support the Van Atta effect or the mirror effect depending on the length of the transmission lines and the angle of incidence. In some cases there is an increase or a decrease in the reflection of about 100 per cei

6. THE OPTIMUM TRANSMISSION LINE LENGTH.

So far it is seen that if we choose the length of the transmission lines arbitrary we cannot expect the reflector to act as stated in the patent description. For the reflector with a half wavelength between adjacent elements numerical calculations have been made in order to find the optimum transmission line length, i.e. the length at which the reflector is as much as possible in accordance with the patent description.

A number of radiation patterns have been calculated for varying values of ℓ and ϕ_i, and by comparison and selection an atempt has been made to find the optimum length. It turned out that two length, ℓ_m and ℓ_g, are possible depending on how we want the reflector to work.

At ℓ_m the reflection has a maximum as close as possible to the direction of arrival for all angles of incidence. This happens when ℓ is 0.28λ . When ℓ is in the interval 0.20λ to 0.35λ the radiation pattern does not differ much from that shown in fig. 11 for $\ell = 0.28\lambda$. From this it is seen that the choice of ℓ_m is not critical.

At ℓ_g the reflection opposite the direction of arrival is as large as possible. This happens when ℓ is 0.64λ and the corresponding radiation pattern is shown in fig. 12. In this case the direction of maximum reflection may diverge by up to 30° from the direction opposite the direction of incidence. At this length the minor effect is more pronounced than the Van Atta effect.

7. CONCLUSION.

The theoretical and numerical investigation shows that a passive linear Van Atta reflector consisting of 4 half wave dipoles does not work precisely as claimed in Van Atta's patent description, that is, it does not transmit an incident electromagnetic wave back in the direction from whence it came for arbitrary angles of incidence and arbitrary lengths of the transmission lines. This is due to the scattering effect of the dipoles and to the coupling between the dipoles.

Among other results it has been shown that for some lengths of the transmission lines and for some angles of incidence there is no reflection at all.

In the numerical calculations it appears that the induction causes the radiation pattern to be asymmetrical and may support the reflection opposite the direction of incidence. The length of the transmission lines at which the reflection is as much as possible in agreement with the patent description has been found. Even at this length the reflector has the mirror effect, so that if it is possible to use the reflector with a Van Atta effect, it is also possible to use the reflector as a mirror.

8. APPENDIX

It will be proved that the scattered energy and the energy received by a matched antenna are equal, if the power scattered by the open-circuited antenna is much smaller than that scattered by the matched antenna. If this assumption holds the echo area of the antenna is (3)

$$\sigma = \frac{4AGR_{an}^2}{|Z_{an}+Z_4|^2} \text{,} \qquad\qquad 1A.$$

Where Z_{an} and Z_4 is the imput and load impedance of the antenna, $R_{an} =$ Re(Z_{an}), A is the effective areal, and G the power gain.

If the antenna is matched we get from (1A)

$$\sigma = AG \qquad\qquad 2A.$$

Using the definition of σ, A and G the scattered energy and received energy is found to be the same and equal to AS_i, where S_i is the intensity of the incident field.

For a half-wave dipole the backscattering crosssection is $0.2\lambda^2$ and $0.01\lambda^2$ when the dipole is matched (4) and open circuit (5), respectively.

This shows that the consideration in this report are valid when the antennas in the reflector are half-wave dipoles.

9. LITERATURE

(1) L.C. Van Attu, "Electromagnetic reflector". U.S. Patent no. 2908002, serial no. 514040, (Oct. 6. 1959).

(2) Samuel Silver, "Microwave antenna theory and design". M.I.T. Rad. Lab. Ser. Vol. 12, Sec. 3.16, p. 98. 1949.

(3) R.F. Harrington, "Electromagnetic scattering by antennas". IEEE Trans. on Antennas and Propagation. Vol. AP-11, No. 5, p. 595. Sept. 1963.

(4) Yuen-Ying Hu, "Back scattering cross section of a center-loaded cylindrical antenna". IRE, Trans. on Antennas and Propagation, Vol. AP-6, pp. 140-148. Jan. 58.

(5) Roger F. Harrington, "Small resonant scatterers and their use for field measurements". IRE, Trans. Micr. Theory and Tech., Vol. 10, pp. 165-174. May 1962.

$-Z_i$
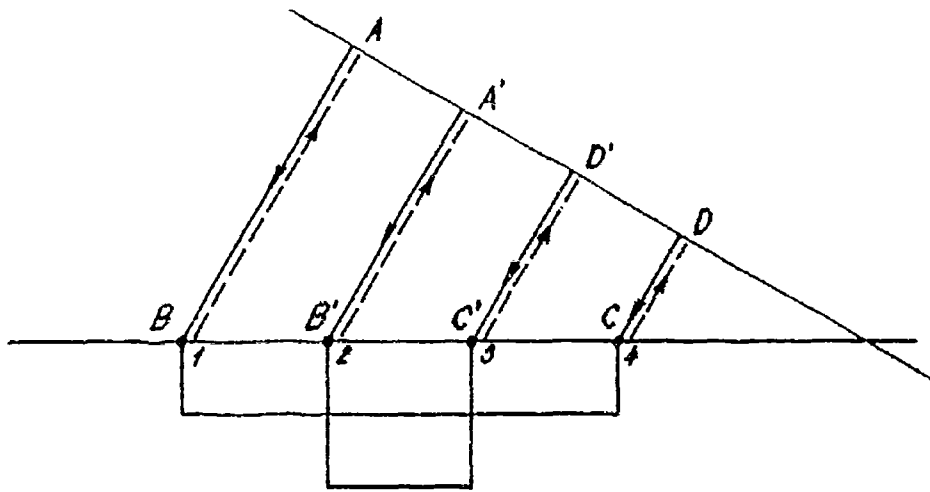
fig 1. The Van Atta effect.
The paths ABCD and A'B'C'D' are equal.
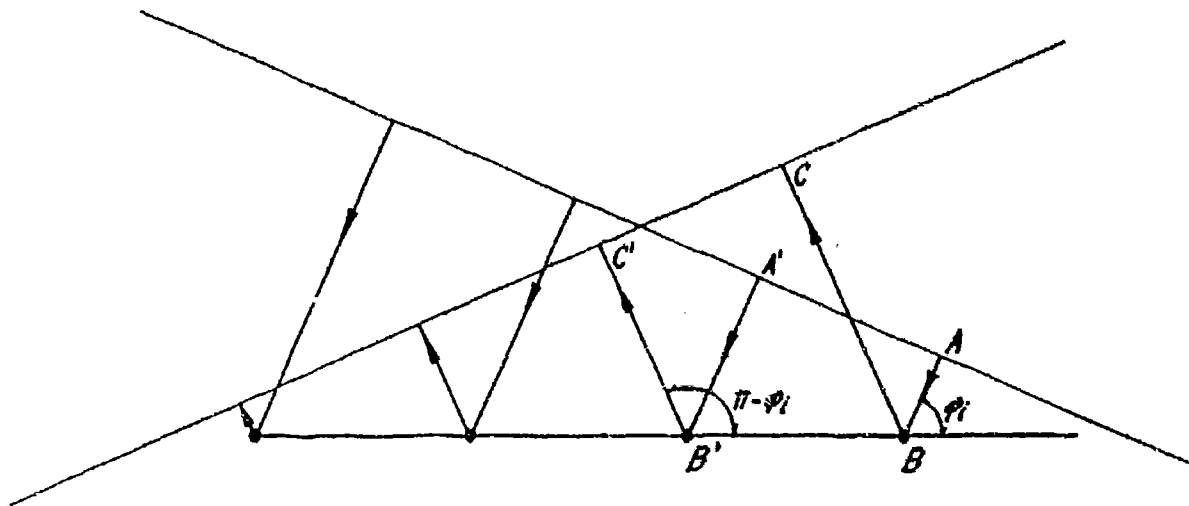
fig 2. The mirror effect.
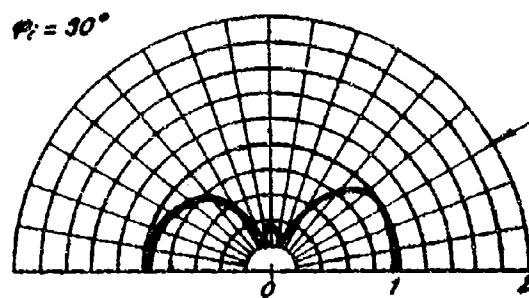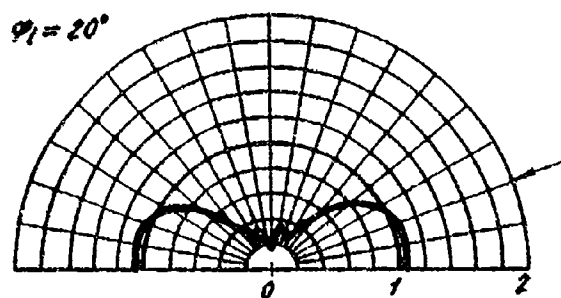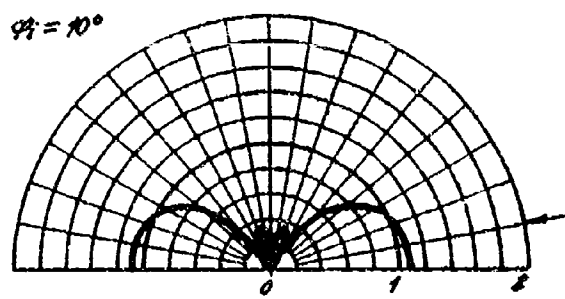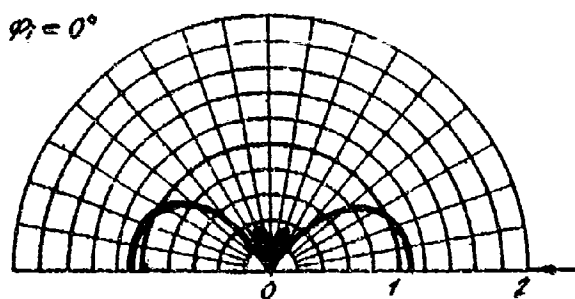The paths ABC and A'B'C' are equal.
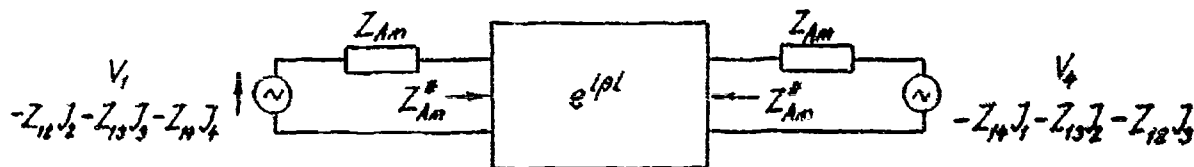
fig 3. Equivalent diagram for dipole 1 and 4.
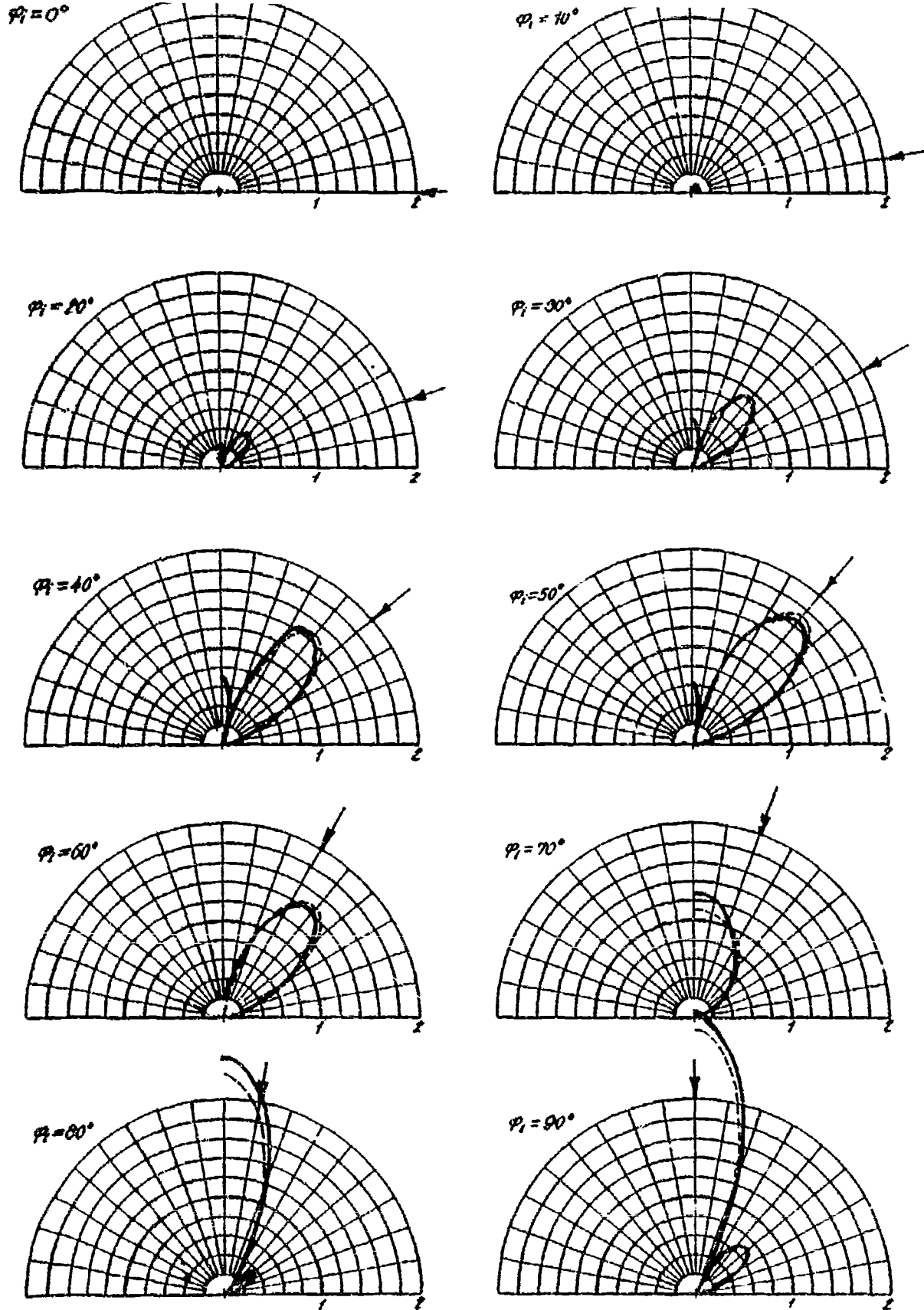
Fig 4. Radiation patterns when $l = 0.5\,\lambda$.

- - - - mutual impedances neglected.

———— mutual impedances taken into account.
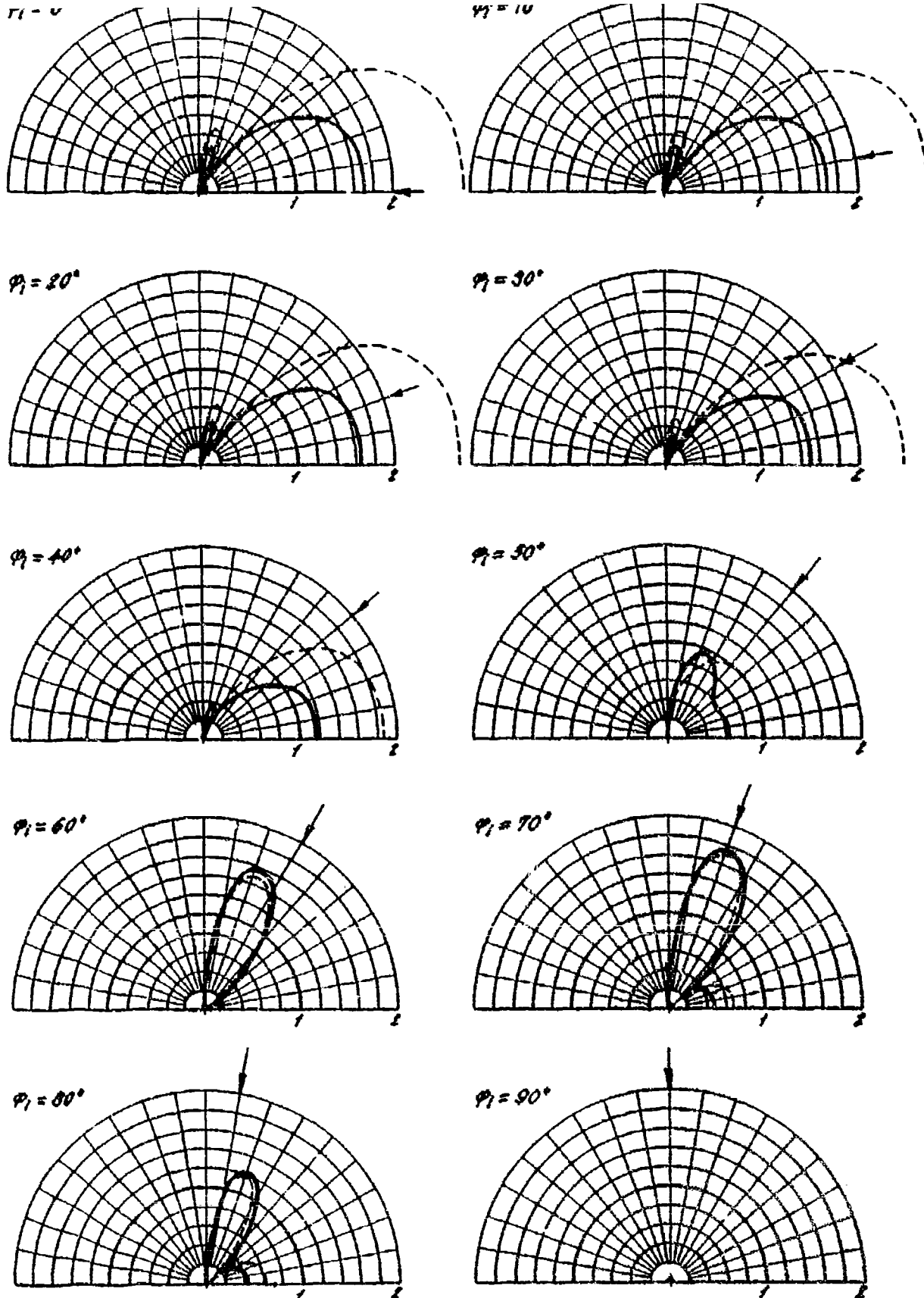
Fig 5. Radiation patterns when l = 1 λ.

---- mutual impedances neglected.

—— mutual impedances taken into account.
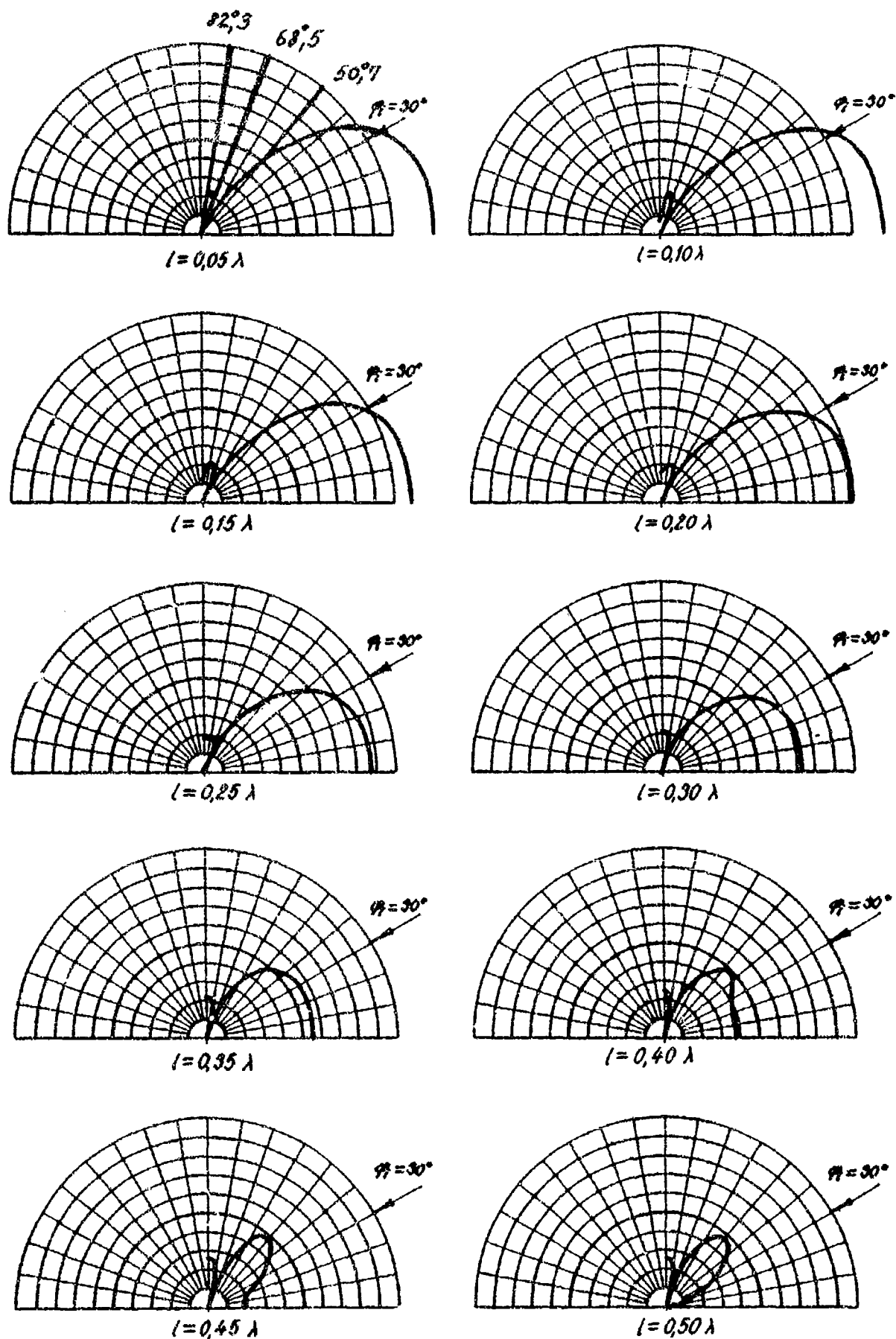
Fig 6. Radiation patterns when $\varphi_1 = 30°$;
mutual impedances neglected.
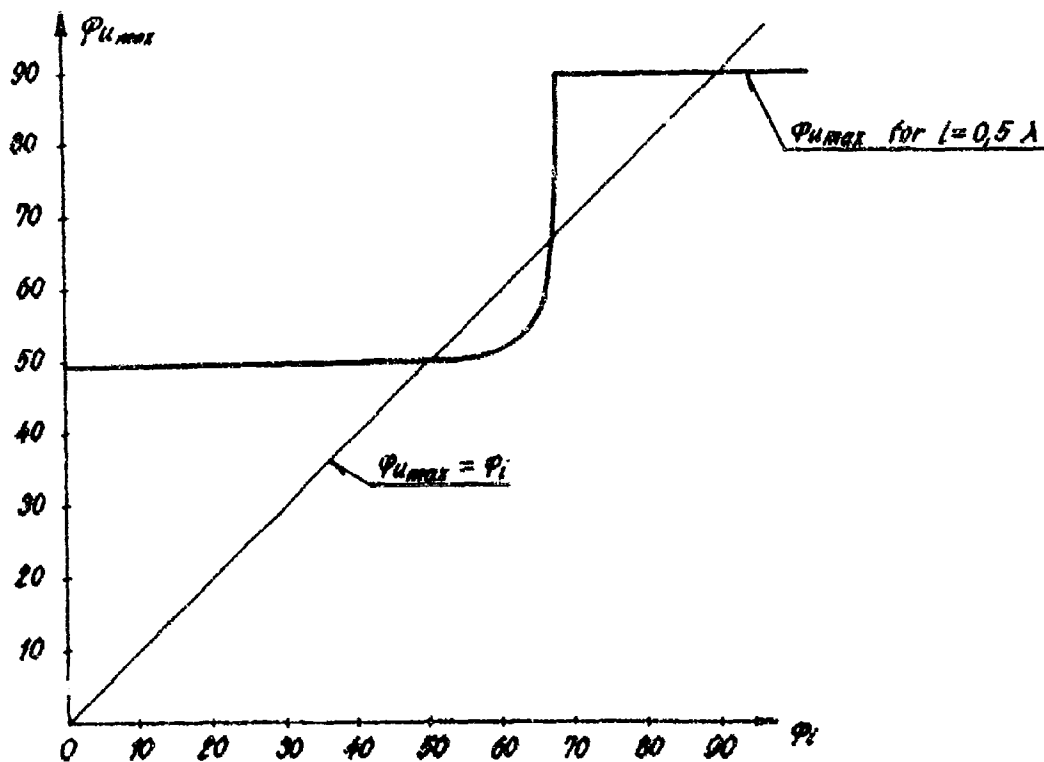
fig 7.

φ_u max versus φ_i, when $l = 0.5\,\lambda$.
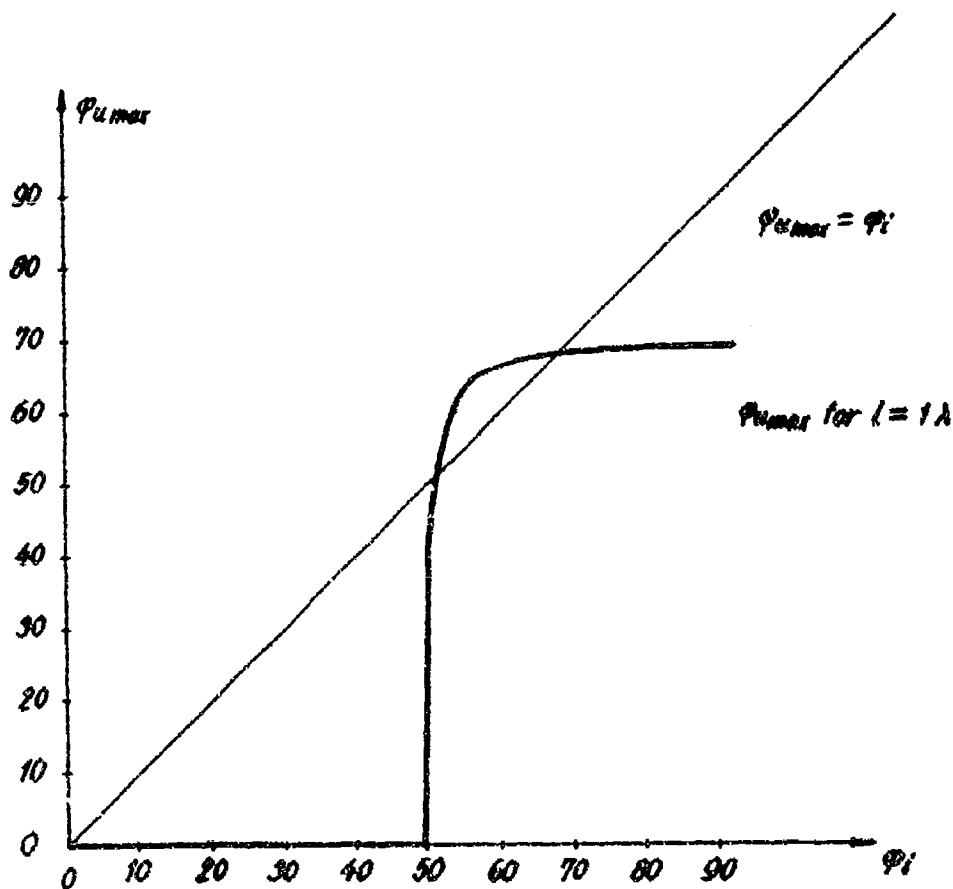
fig 8.

φ_u max versus φ_i, when $l = 1\,\lambda$.
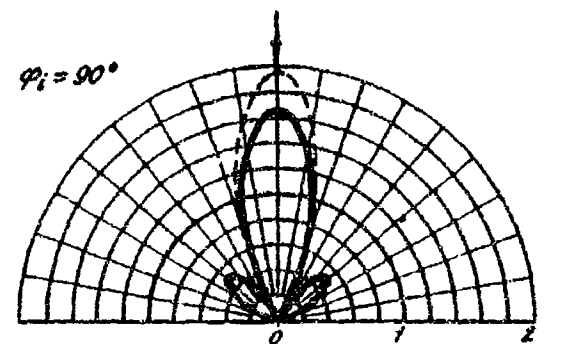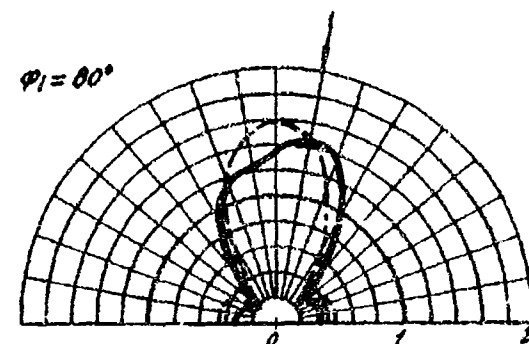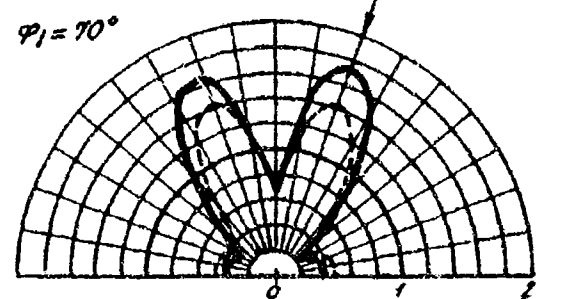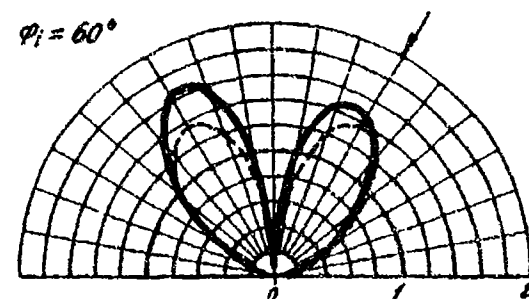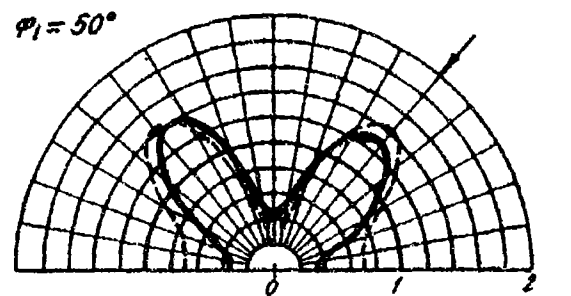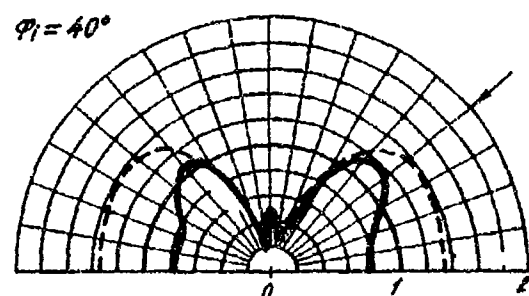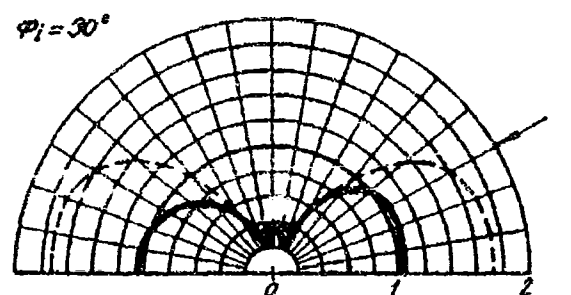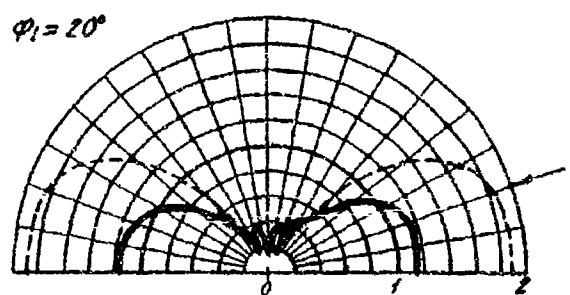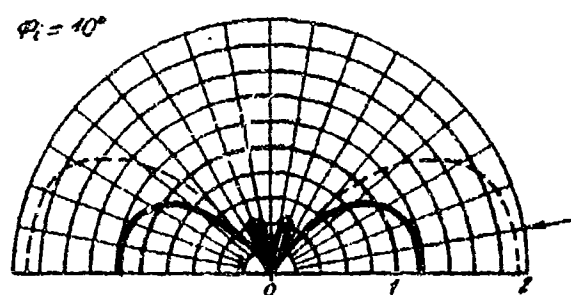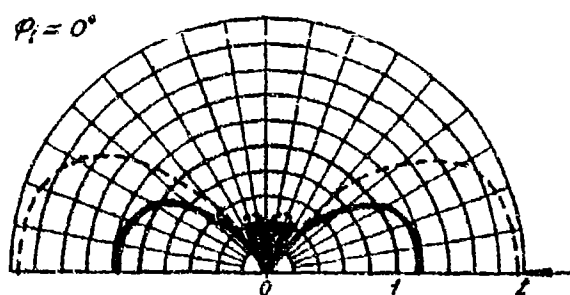
Fig 9. Radiation patterns when $l = 0.25\,\lambda$.
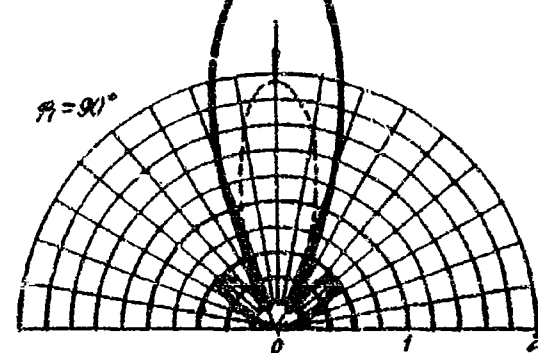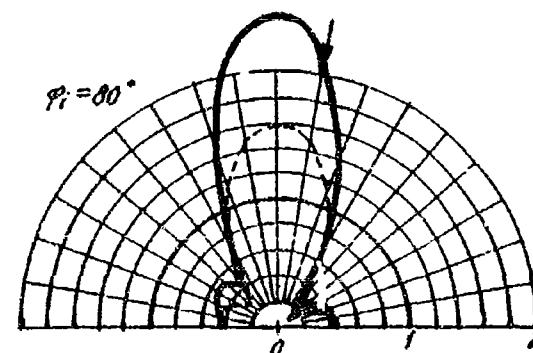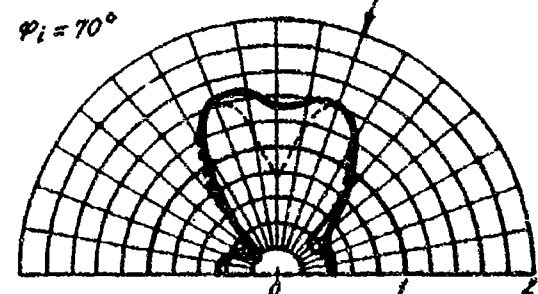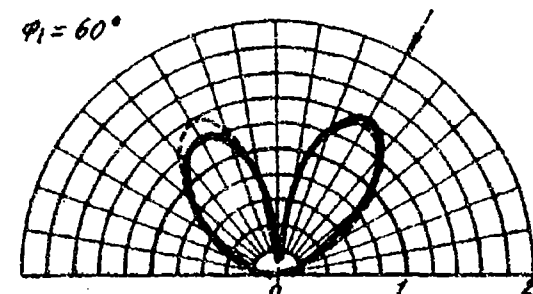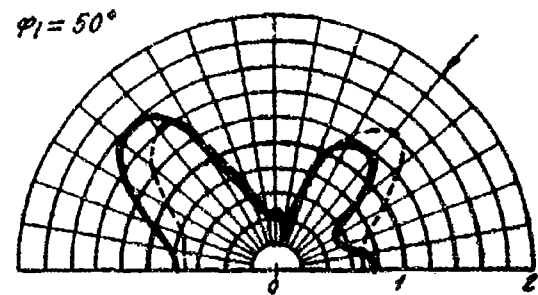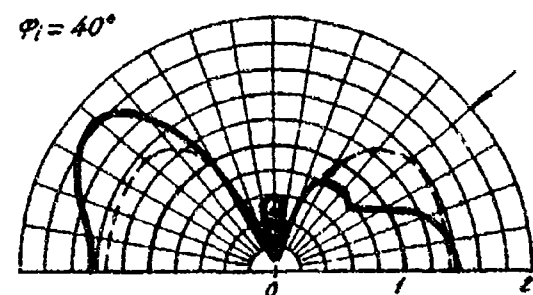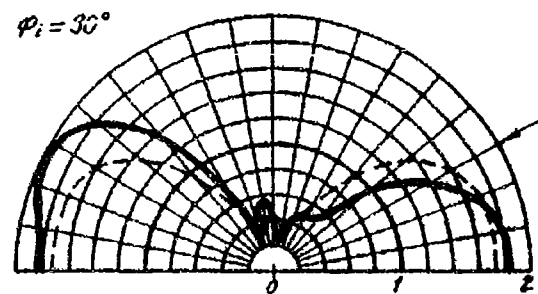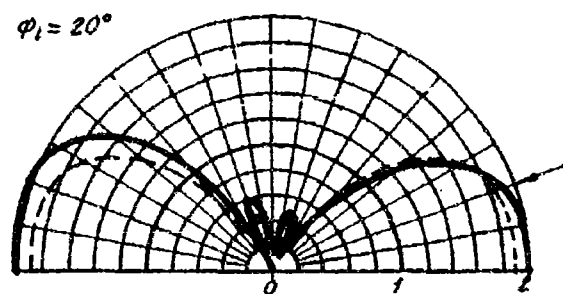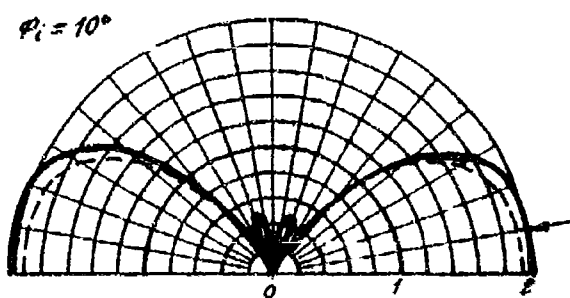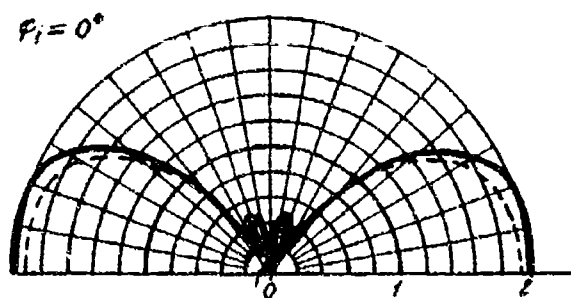
Fig 10. Radiation patterns when $l = 0.75 \lambda$.

$\varphi_l = 0°$ $\varphi_l = 10°$

$\varphi_l = 20°$ $\varphi_l = 30°$

$\varphi_l = 40°$ $\varphi_l = 50°$

$\varphi_l = 60°$ $\varphi_l = 70°$

$\varphi_l = 80°$ $\varphi_l = 90°$
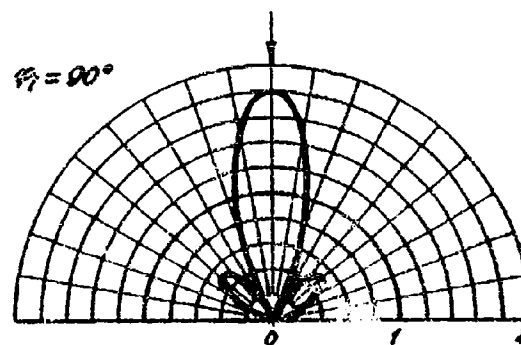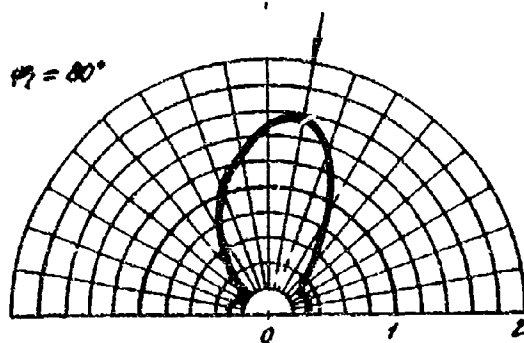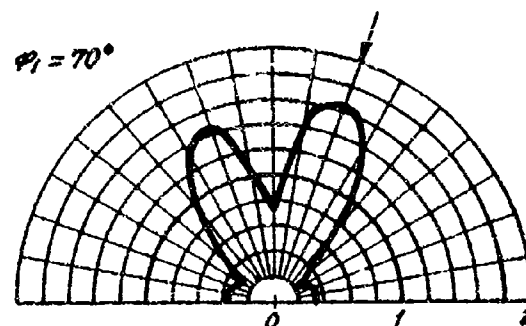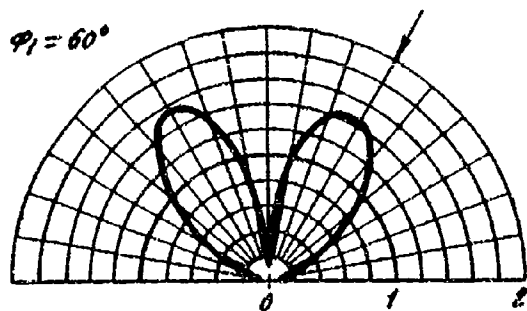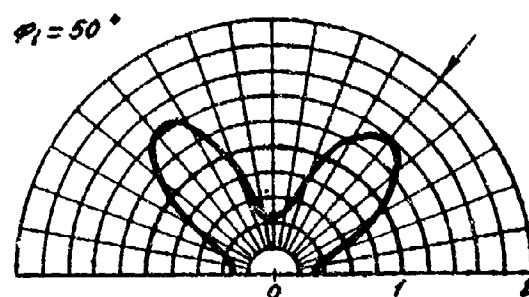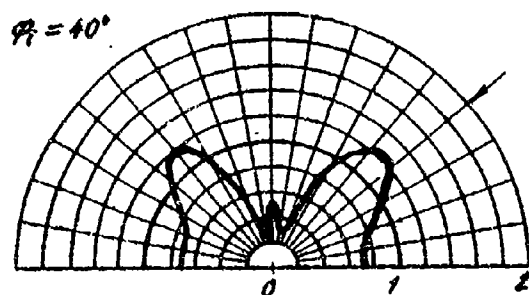
Fig 11. Radiation patterns when $l = l_m = 0.28\lambda$.

$\varphi_i = 0°$ $\varphi_i = 10°$
$\varphi_i = 20°$ $\varphi_i = 30°$
$\varphi_i = 40°$ $\varphi_i = 50°$
$\varphi_i = 60°$ $\varphi_i = 70°$
$\varphi_i = 80°$ $\varphi_i = 90°$

Fig. 12. Radiation pattern